Section 4.5 Integration by Substitution

Recap- u is usually a function of x

1. Power Rule (where n≠1) $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C$
2. Natural Logarithm Rule $\int \frac{1}{u}du = \ln|u| + C$
3. Exponential Rule $\int e^u\,du = e^u + C$

Example 1: Evaluate
$\int (3x-4)^7 dx$

$u = 3x-4$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$\int u^7 \frac{du}{3}$

$\int \frac{1}{3}u^7\,du$

$\frac{1}{3}\cdot\frac{1}{7+1}u^{7+1} + C$

$\frac{1}{24}u^8 + C$

$\frac{1}{24}(3x-4)^8 + C$

Example 2: Evaluate
a. $\int 3x^2(x^3+1)^{10}dx$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3x^2} = dx$

$\int 3x^2 \cdot u^{10} \cdot \frac{du}{3x^2}$

$\int u^{10}\,du$

$\frac{1}{11}u^{11} + C$

$\frac{1}{11}(x^3+1)^{11} + C$

b. $\int \frac{2x}{1+x^2}dx$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int \frac{2x}{u} \cdot \frac{du}{2x}$

$\int \frac{1}{u}\, du$

$\ln|u| + C$

$\ln|1+x^2| + C$ OR $\ln(1+x^2) + C$ since $1+x^2 > 0$

c. $\int \frac{2x}{(1+x^2)^5}dx$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int \frac{2x}{u^5} \cdot \frac{du}{2x}$

$\int \frac{1}{u^5}\, du$

$\int u^{-5}\, du$

$\frac{1}{-5+1} u^{-5+1} + C$

$-\frac{1}{4}u^{-4} + C = -\frac{1}{4}(1+x^2)^{-4} + C$ OR $\frac{-1}{4(1+x^2)^4} + C$

d. $\int xe^{x^2}dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int x \cdot e^u \cdot \frac{du}{2x}$

$\int \frac{1}{2} e^u\, du$

$\frac{1}{2}e^u + C$

$= \frac{1}{2}e^{x^2} + C$

e. $\int \frac{1}{x+3} dx$

$u = x+3$

$\frac{du}{dx} = 1$

$du = dx$

$\int \frac{1}{u} du$

$\ln|u| + C$

$\ln|x+3| + C$

Example 4: Evaluate

$$\int_0^2 (x+1)(x^2+2x+3)^4 dx$$

$u = x^2+2x+3$

$\frac{du}{dx} = 2x+2$

$\frac{du}{dx} = 2(x+1)$

$\frac{du}{2(x+1)} = dx$

$\int (x+1)\, u^4 \frac{du}{2(x+1)}$

$\int \frac{1}{2} u^4 du$

$\frac{1}{2} \cdot \frac{1}{5} u^5$

$\frac{1}{10}(x^2+2x+3)^5 \Big|_0^2$

$\frac{1}{10}(2^2+2\cdot 2+3)^5 - \frac{1}{10}(0^2+2\cdot 0+3)^5$

$\frac{1}{10}(11)^5 - \frac{1}{10}(3)^5$

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