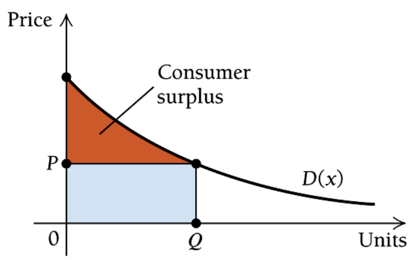
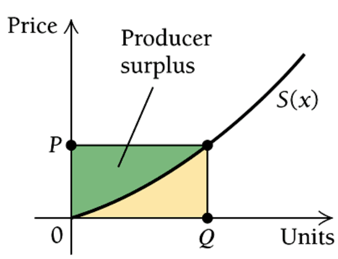
Section 5.1 Consumer Surplus and Producer Surplus, Equilibrium

It is convenient to consider demand and supply as functions of price.



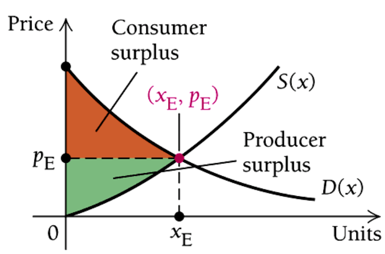
The consumer's supply curve is the graph of $p = D(x)$, which is the price per unit, p, that a consumer is willing to pay for x units. It is usually a decreasing function since the consumer expects to pay less per unit for large quantities.

Suppose that $p = D(x)$ describes the demand function for a product. Then, the **consumer surplus** for Q units of the product, at price P per unit, is $\int_0^Q D(x)dx - QP$.



The producer's supply curve is the graph of $p = S(x)$, which is the price per unit, p, that a producer is willing to accept for selling x units. It is usually an increasing function since a higher price per unit is an incentive for the producer to make more units available for sale.

Suppose that $p = S(x)$ describes the demand function for a product. Then, the **producer surplus** for Q units of the product, at price P per unit, is $QP - \int_0^Q S(x)dx$.



The equilibrium point, (x_E, p_E), is the point at which the supply and demand curves intersect.

It is that point at which sellers and buyers come together and purchases and sales actually occur.

Example 1: Given $D(x) = (x-5)^2$ and $S(x) = x^2 + x + 3$, find each of the following:

a. The equilibrium point.

b. The consumer surplus at the equilibrium point.

c. The producer surplus at the equilibrium point.

Example 2: Given $D(x) = x^2 - 6x + 16$ and $S(x) = \frac{1}{3}x^2 + \frac{4}{3}x + 4$, find each of the following. Assume x≤5.

a. The equilibrium point.
b. The consumer surplus at the equilibrium point.
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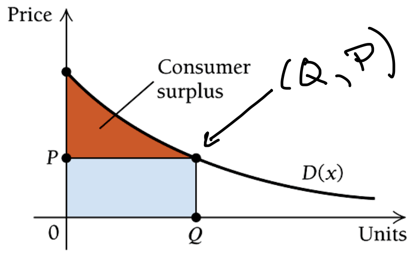

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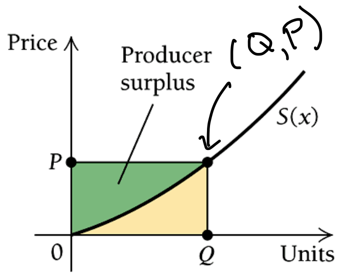

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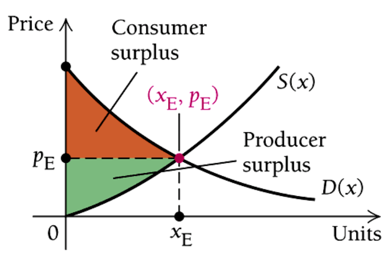

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a. The equilibrium point.

b. The consumer surplus at the equilibrium point.

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a. $D(x) = S(x)$

$(x-5)^2 = x^2 + x + 3$

$x^2 - 10x + 25 = x^2 + x + 3$ subtract x^2

$-10x + 25 = x + 3$ subtract x

$-11x + 25 = 3$ subtract 25

$-11x = -22$ divide by -11

$x = \frac{-22}{-11} = 2$

$D(2) = (2-5)^2 = (-3)^2 = 9$

Equilibrium Point

$(2, 9)$

Q P

b. Consumer Surplus

$$\int_0^Q D(x)\,dx - QP = \int_0^2 (x-5)^2\,dx - 2\cdot 9$$

$$= \int_0^2 (x^2-10x+25)\,dx - 18$$

$$= \frac{1}{3}x^3 - 5x^2 + 25x \Big|_0^2 - 18$$

$$= \left(\frac{1}{3}\cdot 2^3 - 5\cdot 2^2 + 25\cdot 2\right) - \left(\frac{1}{3}\cdot 0^3 - 5\cdot 0^2 + 25\cdot 0\right) - 18$$

$$= \frac{8}{3} - 20 + 50 - 0 - 18$$

$$= \$14.67$$

c. Producer Surplus

$$QP - \int_0^Q S(x)\,dx =$$

$$2\cdot 9 - \int_0^2 (x^2+x+3)\,dx =$$

$$18 - \left[\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3x\right)\Big|_0^2\right]$$

$$18 - \left[\left(\frac{1}{3}\cdot 2^3 + \frac{1}{2}\cdot 2^2 + 3\cdot 2\right) - \left(\frac{1}{3}\cdot 0^3 + \frac{1}{2}\cdot 0^2 + 3\cdot 0\right)\right]$$

$$18 - \left[\left(\frac{8}{3} + 2 + 6\right) - 0\right]$$

$$18 - \frac{8}{2} - 2 - 6 = \$7.33$$

Example 2: Given $D(x) = x^2 - 6x + 16$ and $S(x) = \frac{1}{3}x^2 + \frac{4}{3}x + 4$, find each of the following. Assume x≤5.

a. The equilibrium point.
b. The consumer surplus at the equilibrium point.
c. The producer surplus at the equilibrium point.

a. $x^2 - 6x + 16 = \frac{1}{3}x^2 + \frac{4}{3}x + 4$ mult. by 3

$3x^2 - 18x + 48 = x^2 + 4x + 12$

divide by 2

$\frac{2x^2}{2} - \frac{22x}{2} + \frac{36}{2} = \frac{0}{2}$

factor

$x^2 - 11x + 18 = 0$

$(x-2)(x-9) = 0$

$x - 2 = 0$ $\quad$ $x - 9 = 0$

$x = 2$ $\quad$ $x = 9$

Eq Point $(2, 8)$

$D(2) = 2^2 - 6 \cdot 2 + 16 = 8$

b. consumer surplus

$\int_0^Q D(x)\,dx - QP =$

$\int_0^2 (x^2 - 6x + 16)\,dx - 2 \cdot 8 =$

$\left(\frac{1}{3}x^3 - 3x^2 + 16x\right)\Big|_0^2 - 16$

$\left(\frac{1}{3} \cdot 2^3 - 3 \cdot 2^2 + 16 \cdot 2\right) - \left(\frac{1}{3} \cdot 0^3 - 3 \cdot 0^2 + 16 \cdot 0\right) - 16$

$\frac{8}{3} - 12 + 32 - 0 - 16$ $= \$6.67$

C. Producer surplus

$$QP - \int_0^Q S(x)\,dx =$$

$$2 \cdot 8 - \int_0^2 \left(\frac{1}{3}x^2 + \frac{4}{3}x + 4\right) dx$$

$$16 - \left(\frac{1}{9}x^3 + \frac{2}{3}x^2 + 4x\right)\Big|_0^2$$

$$16 - \left[\left(\frac{1}{9}\cdot 2^3 + \frac{2}{3}\cdot 2^2 + 4\cdot 2\right) - \left(\frac{1}{9}\cdot 0^3 + \frac{2}{3}\cdot 0^2 + 4\cdot 0\right)\right]$$

$$16 - \left[\left(\frac{8}{9} + \frac{8}{3} + 8\right) - 0\right]$$

$$16 - \frac{8}{9} - \frac{8}{3} - 8 = \$4.44$$