

Section 6.1 Functions of Several Variables

A function of two variables, f , assigns to each input pair, (x, y) , exactly one output number, $f(x, y)$.

Example 1: Suppose $P(x, y) = 4x + 6y$. Find $P(25, 10)$.

Example 2: A company's cost function is given by $C(x, y) = 6.5x + 7.25y$. Find $C(10, 15)$.

Example 3: The total cost to a company, in thousands of dollars, is given by

$$C(x, y, z, w) = 4x^2 + 5y + z - \ln(w + 1)$$

where x dollars is spent for labor, y dollars for raw materials, z dollars for advertising, and w dollars for machinery. Find $C(3, 2, 0, 10)$.

Example 4: Large purchases are often financed with an amortized loan. Borrowers like to know how much they can expect to pay per month for every thousand dollars borrowed. The monthly payment P depends on the annual percentage rate (APR), r , and the term of the loan, t (in years). The function P of the two variables r and t is given by

$$P(r, t) = \frac{1000r \left(1 + \frac{r}{12}\right)^{12t}}{12 \left(1 + \frac{r}{12}\right)^{12t} - 12}$$

How much per month can a borrower expect to pay per thousand dollars borrowed at an APR of 4.5% for a 6 year term?

Example 5: The formula in the previous example is used to generate a table of payments that shows borrowers the combined effects of r , the APR, and the term t (in years).

Monthly Payment per \$1000 Borrowed

Annual Percentage Rate, r	Term, t (in years)				
	4	5	6	7	8
0.05	\$23.03	\$18.87	\$16.10	\$14.13	\$12.66
0.055	\$23.26	\$19.10	\$16.34	\$14.37	\$12.90
0.06	\$23.49	\$19.33	\$16.57	\$14.61	\$13.14
0.065	\$23.71	\$19.57	\$16.81	\$14.85	\$13.39
0.07	\$23.95	\$19.80	\$17.05	\$15.09	\$13.63
0.075	\$24.18	\$20.04	\$17.29	\$15.34	\$13.88

- How much per month can a borrower expect to pay per thousand dollars borrowed at an APR of 5.5% for a 6 year term?
- Suppose Sherry can borrow \$3000 at an APR of 5.5% for a 7-yr term. Find her monthly payment.
- Sherry is also considering borrowing the \$3000 from another lender at an APR of 6.5% for a 5-yr term. Find her monthly payment.
- Assume that Sherry makes the minimum payment each month for the entire term of the loan. Calculate her total payments for parts b and c above. Which option costs Sherry less overall?

Section 6.1 Functions of Several Variables

A function of two variables, f , assigns to each input pair, (x, y) , exactly one output number, $f(x, y)$.

Example 1: Suppose $P(x, y) = 4x + 6y$. Find $P(25, 10)$.

$$P(25, 10) = 4(25) + 6(10) = 160$$

Example 2: A company's cost function is given by $C(x, y) = 6.5x + 7.25y$. Find $C(10, 15)$.

$$C(10, 15) = 6.5(10) + 7.25(15) = 173.75$$

Example 3: The total cost to a company, in thousands of dollars, is given by

$$C(x, y, z, w) = 4x^2 + 5y + z - \ln(w + 1)$$

where x dollars is spent for labor, y dollars for raw materials, z dollars for advertising, and w dollars for machinery. Find $C(3, 2, 0, 10)$.

$$\begin{aligned} C(3, 2, 0, 10) &= 4(3)^2 + 5(2) + 0 - \ln(10 + 1) \\ &= 36 + 10 - \ln(11) \\ &= 43.6 \text{ thousands} \rightarrow \$43,600 \end{aligned}$$

Example 4: Large purchases are often financed with an amortized loan. Borrowers like to know how much they can expect to pay per month for every thousand dollars borrowed. The monthly payment P depends on the annual percentage rate (APR), r , and the term of the loan, t (in years). The function P of the two variables r and t is given by

$$P(r, t) = \frac{1000r \left(1 + \frac{r}{12}\right)^{12t}}{12 \left(1 + \frac{r}{12}\right)^{12t} - 12}$$

How much per month can a borrower expect to pay per thousand dollars borrowed at an APR of 4.5% for a 6 year term?

$$P(.045, 6) = \frac{1000 \cdot .045 \left(1 + \frac{.045}{12}\right)^{12 \cdot 6}}{12 \left(1 + \frac{.045}{12}\right)^{12 \cdot 6} - 12}$$

$$= \$15.87$$

The monthly payment is \$15.87 per thousand dollars borrowed

Example 5: The formula in the previous example is used to generate a table of payments that shows borrowers the combined effects of r , the APR, and the term t (in years).

Monthly Payment per \$1000 Borrowed

Annual Percentage Rate, r	Term, t (in years)				
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a. How much per month can a borrower expect to pay per thousand dollars borrowed at an APR of 5.5% for a 6 year term?

$$\$16.34$$

b. Suppose Sherry can borrow \$3000 at an APR of 5.5% for a 7-yr term. Find her monthly payment.

$$\$14.37 \cdot 3 = \$43.11$$

three thousand \nearrow

c. Sherry is also considering borrowing the \$3000 from another lender at an APR of 6.5% for a 5-yr term. Find her monthly payment.

$$\$19.57 \cdot 3 = \$58.71$$

d. Assume that Sherry makes the minimum payment each month for the entire term of the loan. Calculate her total payments for parts b and c above. Which option costs Sherry less overall?

$$5.5\% , 7\text{yr}$$

$$\$43.11 (7 \cdot 12) = \$3621.24$$

option b

$$6.5\% , 5\text{yr}$$

$$\$58.71 (5 \cdot 12) = \$3522.60$$

option c ✓
lower total payment