Section 6.2 Partial Derivatives

Notation $\frac{\partial z}{\partial x}$ $\frac{\partial f}{\partial x}$ f_x all mean partial derivative with respect to x

When you are taking a partial derivative, consider the other variables as fixed constants

Example 1: Consider the function $z = f(x, y) = x^2y^3 + xy + 4y^2$

a. Find $\frac{\partial z}{\partial x}$, the partial derivative of z with respect to x and consider y as a fixed constant.

b. Find $\frac{\partial z}{\partial y}$, the partial derivative of z with respect to y and consider x as a fixed constant.

Example 2: For $w = x^2 - xy + y^2 + 2yz + 2z^2 + z$, find the following.

a. $\frac{\partial w}{\partial x}$

b. $\frac{\partial w}{\partial y}$

c. $\frac{\partial w}{\partial z}$

Example 3: For $f(x, y) = 3x^2y + xy^2$, find $f_x(x, y)$ at $(2, -3)$.

Example 4: For $f(x, y) = e^{xy} + y \ln x$, find f_x and f_y.

Example 5: For $f(x, y) = \sqrt{x^2 - y^2}$, find $f_y(-3, -2)$

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Example 1: Consider the function $z = f(x, y) = x^2y^3 + xy + 4y^2$

a. Find $\frac{\partial z}{\partial x}$, the partial derivative of z with respect to x and consider y as a fixed constant.

$z = x^2y^3 + xy + 4y^2$

$\frac{dz}{dx} = y^3 \cdot 2x + y \cdot 1 + 0 = 2xy^3 + y$

b. Find $\frac{\partial z}{\partial y}$, the partial derivative of z with respect to y and consider x as a fixed constant.

$z = x^2y^3 + xy + 4y^2$

$\frac{dz}{dy} = x^2 \cdot 3y^2 + x \cdot 1 + 8y = 3x^2y^2 + x + 8y$

Example 2: For $w = x^2 - xy + y^2 + 2yz + 2z^2 + z$, find the following.

a. $\frac{\partial w}{\partial x} = 2x - y \cdot 1 + 0 + 0 + 0 + 0 = 2x - y$

b. $\frac{\partial w}{\partial y} = 0 - x \cdot 1 + 2y + 2z \cdot 1 + 0 + 0 = -x + 2y + 2z$

c. $\frac{\partial w}{\partial z} = 0 - 0 + 0 + 2y \cdot 1 + 4z + 1 = 2y + 4z + 1$

Example 3: For $f(x,y) = 3x^2y + xy^2$, find $f_x(x,y)$ at $(2,-3)$.

$x \quad y$

$$f_x = 3y \cdot 2x + y^2 \cdot 1 = 6xy + y^2$$

$$f_x(2,-3) = 6 \cdot 2 \cdot (-3) + (-3)^2 = -36 + 9 = -27$$

Example 4: For $f(x,y) = e^{xy} + y \ln x$, find f_x and f_y.

$$f_x = e^{xy} \cdot y + y \cdot \frac{1}{x} = ye^{xy} + \frac{y}{x}$$

$$f_y = e^{xy} \cdot x + \ln x \cdot 1 = xe^{xy} + \ln x$$

Example 5: For $f(x,y) = \sqrt{x^2 - y^2}$, find $f_y(-3,-2)$

$$f(x,y) = (x^2 - y^2)^{1/2}$$

$$f_y = \frac{1}{2}(x^2 - y^2)^{-1/2}(0 - 2y) = -y(x^2 - y^2)^{-1/2} = \frac{-y}{\sqrt{x^2 - y^2}}$$

$$f_y(-3,-2) = \frac{-(-2)}{\sqrt{(-3)^2 - (-2)^2}} = \frac{2}{\sqrt{9-4}} = \frac{2}{\sqrt{5}}$$